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STABILITY OF CIRCULAR PLATES FROM AGEING VISCOELASTIC MATERIAL*

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Stability conditions are obtained for circular plates of an inhomogeneously-ageing viscoelastic material for an arbitrary creep kernel and different methods of plate support. Stability in an infinite time interval corresponds to determination of the Lyapunov stability, and in a finite interval, Chetayev stability.

1. Formulation of the problem. Consider the axisymmetric deformation of a circular plate of constant thickness h and radius R. We introduce a cylindrical $r\varphi z$ coordinate system whose origin is at the centre of the plate middle plane in the undeformed state, while the z-axis is perpendicular to this plane. At a time $t=0$ an external load is applied to the plate. We denote the age of the plate material at the point r at the time of external load application by $\rho(r)$. The function $\rho(r)$ is piecewise-continuous and bounded.

The stress σ_{ij} and strain ε_{ij} tensor components $(i, j = r, \varphi, z)$ are connected by the relationships

$$\begin{gathered}e_{ij} = (1+\nu)(I+L)s_{ij}/E,\quad \varepsilon = (1-2\nu)\sigma/E\\ s_{ij} = E(1+\nu)^{-1}(I-N)e_{ij},\quad \sigma = E(1-2\nu)^{-1}\varepsilon\\ \sigma = (\sigma_{rr}+\sigma_{\varphi\varphi}+\sigma_{zz})/3,\quad \varepsilon = (\varepsilon_{rr}+\varepsilon_{\varphi\varphi}+\varepsilon_{zz})/3\\ e_{ij} = \varepsilon_{ij} - e\delta_{ij},\quad s_{ij} = \sigma_{ij} - \sigma\delta_{ij}\\ Ix = x(t),\quad Lx = \int_0^t l(t+\rho, \tau+\rho)x(\tau)\,d\tau,\\ Nx = \int_0^t n(t+\rho, \tau+\rho)x(\tau)\,d\tau\end{gathered} \tag{1.1}$$

Here E is the constant modulus of elastic instantaneous deformation, ν is the constant Poisson's ratio, δ_{ij} are Kronecker deltas, I is the unit operator, L is the creep operator, N is the relaxation operator, and $l(t,\tau)$ and $n(t,\tau)$ are the creep and relaxation kernels.

The external load applied to the plate consists of a transverse distributed load of intensity $q(r)$ and compressive forces of constant magnitude p.

Let $w(t, r)$ denote the plate deflection at the point r at the time t, w_0 the maximum allowable value of the deflection, and T_0 the first time the deflection reaches the value w_0.

Definition 1. A plate is called Lyapunov stable in an infinite time interval if for any $\varepsilon > 0$ there exists a $\delta(\varepsilon) > 0$ such that the estimate $|w(t,r)| < \varepsilon$ $(t \geqslant 0,\ r \in [0, R])$ follows from the inequality $|q(r)| < \delta$

Definition 2. A plate is called stable in an interval $[0, T]$ if $T < T_0$.

The aim of this paper is to obtain the conditions for the magnitudes of the compressive forces p for which the plate is stable according to Defintions 1 and 2.

2. Governing equations. Suppose an axisymmetric generalized plane state of stress exists in the plate. Then $\sigma_{r\varphi} = 0$ and the quantities σ_{iz} $(i = r, \varphi, z)$ can be neglected. We consequently obtain from (1.1)

$$\begin{gathered}\sigma_{rr} = E(1-\nu^2)^{-1}[(1-\nu)(I-N)\varepsilon_{rr} + \nu(I-K)(\varepsilon_{rr}+\varepsilon_{\varphi\varphi})]\\ \sigma_{\varphi\varphi} = E(1-\nu^2)^{-1}[(1-\nu)(I-N)\varepsilon_{\varphi\varphi} + \nu(I-K)(\varepsilon_{rr}+\varepsilon_{\varphi\varphi})]\\ K = N\{I-(1+\nu)(1-2\nu)(3\nu-3\nu^2)^{-1}[I+(1+\nu)\times(3-3\nu)^{-1}L]^{-1}\}\end{gathered} \tag{2.1}$$

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Let $k(t, \tau)$ denote the kernel of the operator K.
We consider that a function $m(t, \tau)$ exists such that for all $0 \leqslant \tau \leqslant t$, $0 \leqslant r \leqslant R$

$$0 \leqslant n(t+\rho, \tau+\rho),\ k(t+\rho, \tau+\rho) \leqslant m(t, \tau) \tag{2.2}$$

$$|m| = \sup_t \int_0^t m(t, \tau)\, d\tau < 1$$

According to Kirchhoff's hypothesis /2/

$$\varepsilon_{rr} = \vartheta' z,\ \varepsilon_{\varphi\varphi} = r^{-1}\vartheta z,\ \vartheta = -w' \quad (w' = \partial w/\partial r) \tag{2.3}$$

where z is the distance from the middle plane of the plate. By virtue of (2.1) and (2.3), the bending moments are given by the formulas

$$M_r = \int_{-h/2}^{h/2} \sigma_{rr} z\, dz = D[(1-\nu)(I-N)\vartheta' + \nu(I-K)(\vartheta' + r^{-1}\vartheta)] \tag{2.4}$$

$$M_\varphi = \int_{-h/2}^{h/2} \sigma_{\varphi\varphi} z\, dz = D[(1-\nu)(I-N)r^{-1}\vartheta + \nu(I-K)(\vartheta' + r^{-1}\vartheta)]$$

$$D = Eh^3\,[12(1-\nu^2)]^{-1}$$

If the plate deflection equals zero, then a state of stress and strain characterised by the stress tensor σ_{ij}° is realized therein. Let the angles of rotation of the plate elements be small compared with one and let their squares be neglected. The elongations and shears are substantially less than the angles of rotation and they can be neglected /2/. Then the equilibrium equations of a plate element in the bent state have the form /3/

$$(r\sigma_{rr}^\circ)' - \overset{\circ}{\sigma}_{\varphi\varphi} = 0 \tag{2.5}$$

$$(rM_r)' - M_\varphi - rQ_r = 0,\ (rQ_r)' + rq + h(r\sigma_{rr}^\circ w')' = 0 \tag{2.6}$$

$$Q_r = \int_{-h/2}^{h/2} \sigma_{rz}\, dz$$

We give the boundary conditions in the form

$$\vartheta(t, 0) = 0;\ \vartheta(t, R) = 0 \tag{2.7}$$

$$\sigma_{rr}^\circ(t, R) = p \tag{2.8}$$

The first condition in (2.7) means that under symmetric bending the angle of rotation of the normal at the centre of the plate is zero. The second condition in (2.7) corresponds to stiff clamping of the plate edges.

Let Q_0 denote the transverse force acting on unit length of the arc /4/ $rq(r) = [rQ_0(r)]'$. We differentiate the first equation of (2.6) with respect to r and add it to the second. According to (2.3), we obtain

$$[(rM_r)' - M_\varphi]' + rq - h(r\sigma_{rr}^\circ\vartheta)' = 0$$

We substitute the expression for Q_0 into this equality and integrate with respect to r. Replacing the quantities M_r, M_φ in the relationship obtained in conformity with (2.4), we find /5/

$$[r(1-\nu)(I-N)\ \vartheta' + r\nu(I-K)(\vartheta' + r^{-1}\vartheta)]' - [(1-\nu)\times \tag{2.9}$$
$$(I-N)r^{-1}\vartheta + \nu(I-K)\ (\vartheta' + r^{-1}\vartheta)] = D^{-1}\ (hr\sigma_{rr}^\circ\vartheta - rQ_0)$$

Eq.(2.9) with the boundary conditions (2.7) describes the deflection of a circular plate under an arbitrary transverse load.

3. Derivation of the stability conditions.

We introduce the notation /6/

$$\|f\|^2 = \int rf^2\, dr,\quad \|f\|_0^2 = \int rf^4\, dr,\quad \|f\|_H^2 = \int r(f'^2 + r^{-2}f^2)\, dr \tag{3.1}$$

$$\|\sigma\|^2 = \int r(\sigma_{rr}^2 + \sigma_{\varphi\varphi}^2)\, dr,\quad (\sigma, \varepsilon) = \int r(\sigma_{rr}\varepsilon_{rr} + \sigma_{\varphi\varphi}\varepsilon_{\varphi\varphi})\, dr$$

(the integrals are evaluated between the limits 0 and R).

We multiply (2.9) by $\vartheta(t, r)$ and integrate over r between 0 and R. Integrating by parts and taking account of the boundary conditions (2.7), we obtain

$$\|\vartheta\|_H^2 = (1-\nu)\int r(\vartheta' N\vartheta' + r^{-1}\vartheta N r^{-1}\vartheta)\, dr + \tag{3.2}$$
$$\nu\int r(\vartheta' + r^{-1}\vartheta)K(\vartheta' + r^{-1}\vartheta)\, dr + D^{-1}\int r\vartheta Q_0\, dr +$$
$$hD^{-1}\int r(-\sigma_{rr}^\circ)\vartheta^2\, dr \equiv (1-\nu)I_1 + \nu I_2 + I_3 + I_4$$

From the Cauchy inequality and (2.2) we obtain the estimates

$$|I_3| \leqslant D^{-1} \| Q_0 \| \cdot \| \vartheta \|, \quad |I_4| \leqslant hD^{-1} \| \sigma^\circ \| \cdot \| \vartheta \|_0 \tag{3.3}$$

$$|I_1| \leqslant \int_0^t m(t,\tau) \int r [\vartheta'(t,r)\vartheta'(\tau,r) + r^{-2}\vartheta(t,r)\vartheta(\tau,r)]\, dr\, d\tau \leqslant$$

$$\| \vartheta \|_H \int_0^t m(t,\tau) \| \vartheta \|_H \, d\tau$$

$$|I_2| \leqslant \int_0^t m(t,\tau) [\vartheta'(t,r) + r^{-1}\vartheta(t,r)] [\vartheta'(\tau,r) + r^{-1}\vartheta(\tau,r)]\, dr\, d\tau \leqslant$$

$$\| \vartheta \|_H \int_0^t m(t,\tau) \| \vartheta \|_H \, d\tau$$

We find from (3.2) and (3.3)

$$\| \vartheta \|_H^2 \leqslant \| \vartheta \|_H \int_0^t m(t,\tau) \| \vartheta \|_H \, d\tau + hD^{-1} \| \sigma^\circ \| \cdot \| \vartheta \|_0 + D^{-1} \| Q_0 \| \cdot \| \vartheta \| \tag{3.4}$$

We introduce the notation $H\vartheta = -r^{-1}[(r\vartheta')' - r^{-1}\vartheta]$. We set

$$\lambda^2 = \inf_\vartheta [(H\vartheta, \vartheta) \| \vartheta \|^{-2}], \quad \lambda_0 = \inf_\vartheta [(H\vartheta, \vartheta) \| \vartheta \|_0^{-1}] \tag{3.5}$$

(the infimums of the functional are defined over all functions $\vartheta(t, r) \not\equiv 0$ that satisfy boundary conditions (2.7)).

According to /6/, $\lambda > 0$ and the inequality $\lambda_0 > 0$ will be proved below. Since

$$\| \vartheta \|^2 \leqslant \lambda^{-2} \| \vartheta \|_H^2, \quad \| \vartheta \|_0 \leqslant \lambda_0^{-1} \| \vartheta \|_H^2,$$

it follows from (3.4) that

$$(1 - hD^{-1}\lambda_0^{-1} | \sigma^\circ |_s | \vartheta |_s) \leqslant | m | | \vartheta |_s + (\lambda D)^{-1} \| Q_0 \| \tag{3.6}$$

$$| \sigma^\circ |_s = \sup_t \| \sigma^\circ \|, \quad | \vartheta |_s = \sup_\tau \| \vartheta \|_H$$

Theorem 1. Let

$$| \sigma^\circ |_s < D\lambda_0 h^{-1} (1 - | m |) \tag{3.7}$$

Then the plate is stable.

Proof. It follows from inequalities (3.6) and (3.7) that a constant $C_1 > 0$ exists such that $| \vartheta |_s \leqslant C_1 \| Q_0 \|$. The assertion of the theorem follows from this inequality and the estimate

$$| w(t, r) | \leqslant C_2 \| \vartheta \|_H \leqslant C_2 | \vartheta |_s \ (C_2 > 0 \quad \text{is a constant}).$$

As in /1/ it can be proved that the following holds.

Theorem 3.2. Let there be a function $m_0(t, \tau)$ such that uniformly in $t \geqslant t_0$

$$\lim \int_{t_0}^t [\sup_r | n(t+\rho, \tau+\rho) - m_0(t,\tau) | + \sup_r | k(t+\rho, \tau+\rho) - m_0(t,\tau) |]\, d\tau = 0$$

Then the plate is stable for $| \sigma^\circ |_s < D\lambda_0 h^{-1} (1 - | m_0 |)$.

4. Estimates of the critical forces. We estimate $\| \sigma^\circ \|$ in terms of p. To do this $\| \sigma^\circ \|$ is initially estimated in terms of $\| \varepsilon^\circ \|$, then $\| \varepsilon^\circ \|$ in terms of p. To obtain the first estimate we substitute expression (2.1) for the stress tensor components into $\| \sigma^\circ \|$. By a method analogous to that mentioned in Sect.3 we obtain

$$\| \sigma^\circ \| \leqslant E(1-\nu)^{-1} (1 + | m |) | \varepsilon^\circ |_s, \quad | \varepsilon^\circ |_s = \sup_\tau \| \varepsilon^\circ \| \tag{4.1}$$

We now turn to the estimation of $| \varepsilon^\circ |_s$ in terms of p. We denote the radial displacement of points of the plate by $u(t, r)$. We multiply (2.5) by u and integrate with respect to r between 0 and R. Integrating by parts and taking account of (2.8), we will have

$$\int r (\sigma_{rr}^\circ \varepsilon_{rr}^\circ + \overset{\circ}{\sigma}_{\varphi\varphi} \overset{\circ}{\varepsilon}_{\varphi\varphi})\, dr = -pRu(R) \equiv J_2 \quad (\varepsilon_{rr}^\circ = u', \ \overset{\bullet}{\varepsilon}_{\varphi\varphi} = r^{-1}u)$$

Replacing the stress tensor components here by formulas (2.1), we obtain

$$J_1^2 \equiv \int r [(1-\nu)(\varepsilon_{rr}^{\circ 2} + \overset{\circ 2}{\varepsilon}_{\varphi\varphi}) + \nu (\varepsilon_{rr}^\circ + \overset{\bullet}{\varepsilon}_{\varphi\varphi})^2]\, dr = (1-\nu^2) E^{-1} J_2 + \tag{4.2}$$

$$\int r [(1-\nu)(\varepsilon_{rr}^\circ N \varepsilon_{rr}^\circ + \overset{\circ}{\varepsilon}_{\varphi\varphi} N \overset{\bullet}{\varepsilon}_{\varphi\varphi}) + \nu (\varepsilon_{rr}^\circ + \overset{\bullet}{\varepsilon}_{\varphi\varphi}) K (\varepsilon_{rr}^\circ + \overset{\circ}{\varepsilon}_{\varphi\varphi})]\, dr$$

We transform the quantity J_2 in the following manner:

$$J_2 \equiv -pRu(R) = -p\int (ru)' \, dr = -p\int r(\varepsilon_{rr}^\circ + \overset{\circ}{\varepsilon}_{\varphi\varphi}) \, dr$$

Then by the Cauchy inequality it is possible to obtain from (4.2), as in Sect.3, that

$$J_1^2 \leqslant J_1 \int_0^t m(t, \tau) J_1 \, d\tau + (1-\nu^2) E^{-1} pR \, \| \varepsilon^\circ \|$$

Hence

$$(1 - |m|) \, |J_1|_s^2 \leqslant (1-\nu^2) E^{-1} pR \, |\varepsilon^\circ|_s, \quad |J_1|_s = \sup_\tau J_1 \tag{4.3}$$

But

$$J_1^2 \equiv \int r(\varepsilon_{rr}^{\circ 2} + 2\nu\varepsilon_{rr}^\circ \overset{\circ}{\varepsilon}_{\varphi\varphi} + \overset{\circ 2}{\varepsilon}_{\varphi\varphi}) \, dr \geqslant (1-\nu) \, \| \varepsilon^\circ \|^2$$

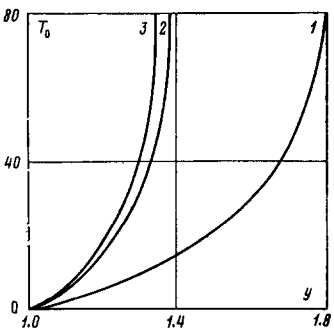
Together with (4.3) and (4.1) this inequality yields the estimate

$$\| \sigma^\circ \| \leqslant P \equiv pR \, (1+\nu) \;\; (1 + |m|) \;\; [(1-\nu) \;\; (1-|m|)]^{-1} \tag{4.4}$$

Using (4.4) for the estimate of I_4 we conclude from (3.3) that the following holds.

Theorem 4.1. If the assumptions of Theorem 3.1 are satisfied, then the plate is stable when $P < D\lambda_0 h^{-1}(1 - |m|)$. If the assumptions of Theorem 3.2 are satisfied, then the plate is stable when

$$P < D\lambda_0 h^{-1} (1 - |m_0|)$$



5. Stability in a finite time interval.

The dependence of the critical time T_0 on the parameters of the problem was investigated numerically for the following values of the quantities /7/: $R = 1$ m, $h = 0.05$ m, $l(t, \tau) = -E\partial/\partial\tau \, [\varphi(\tau) (1 - e^{-\gamma(t-\tau)})]$, $k(t, \tau) \equiv 0$, $\varphi(\tau) = A_0 + A_1 \tau^{-1}$, $E = 3.3 \cdot 10^4$ MPa, $A_0 = 9.75 \cdot 10^{-4}$ MPa^{-1}, $A_1 = 46.2 \cdot 10^{-4}$ MPa^{-1} day, $\nu = 0.3$, $\gamma = 0.03$ day^{-1}, $Q_0 = qr/2$, $q = 0.02$ MPa, $p = 0.1$ MPa.

The age of the plate material is described by a piecewise-constant function equal to $\rho_1 = 3$ days for $0 \leqslant r \leqslant 0.05$ and ρ_2 for $0.05 \leqslant r \leqslant R$.

The dependence of the critical time T_0 on the maximum allowable deflection w_0 is represented in the figure. The dimensionless quantity $y = w_0/w_1$ is plotted along the abscissa axis (w_1 is the value of the deflection corresponding to the elastic problem), and the critical time T_0 in days along the ordinate axis. Curves *1-3* correspond to values of the parameter ρ_2 equal to 3, 22, and 60 days.

A calculation shows thatthe critical time T_0 increases as ρ_2 grows. This dependence is strengthened as w_0 increases.

6. Some remarks.

1°. A constant $C > 0$ exists (/8/, p.84) such that

$$\| \vartheta \| \leqslant C \int r(\vartheta'^2 + r^{-2}\vartheta^2) \, dr$$

Hence the inequality $\lambda_0 > 0$ follows.

2°. The stability conditions obtained hold even for other kinds of plate support. The parameters γ, γ_0 are found from (3.5) with boundary conditions corresponding to the type of support.

3°. Let the plate material be homogeneous $(\rho(r) = \rho_0)$. Then $\sigma_{rr}^\circ = -p$. We let λ_0 denote the minimum eigenvalue of the problem

$$(r\vartheta')' - r^{-1}\vartheta + \lambda\vartheta = 0, \quad \vartheta(0) = \vartheta(R) = 0$$

Theorem 6.1. We assume that

$$p < D\lambda_0 h^{-1}(1 - |m|) \quad (p < D\lambda_0 h^{-1}(1 - |m_0|))$$

Then the homogeneous plate is stable.

For $|m| = 0$ the conditions obtained are in agreement with the stability conditions for an elastic plate /9/.

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A MATHEMATICAL MODEL OF THE PROBLEM OF DIAGNOSING A THERMOELASTIC MEDIUM*

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The diagnosis problem is understood to mean the problem of determining material characteristics by means of information on the physical fields originating therein under the influence of external effects. The problem is from the class of inverse problems of mathematical physics /1/ and is solved using the model of generalized thermomechanics for weakly anisotropic media. As a result of the analysis of wave processes in such a medium, a method is developed for determining the thermoelastic characteristics by means of the temperature and displacement values on the half-space boundary. Examples of calculating specific characteristics are examined.

1. We shall consider the problem of diagnosing a thermoelastic medium within the framework of the model of generalized thermomechanics /2/

$$q_{j,j}+C_\varepsilon\Theta^{\cdot}+T_0\beta_{ij}\varepsilon_{ij}^{\cdot}=0,\quad \tau q_j^{\cdot}+q_j=-K_{ij}\Theta_{,i},\quad \sigma_{ij,j}=\rho u_i^{\cdot\cdot} \tag{1.1}$$
$$\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}),\quad \sigma_{ij}=C_{ijkl}e_{kl}-\beta_{ij}\Theta,\quad i,j,k,l=1,2,3$$

Here C_ε is the specific heat for constant deformation, $\beta_{ij}=C_{ijkl}\alpha_{kl}$, α_{kl} are the coefficients of linear expansion, C_{ijkl} are the isothermal stiffness coefficients of an anisotropic material, K_{ij} are the thermal conductivities, τ is the heat flux relaxation time, ρ is the density (the quantities listed above are functions of the space variables $\mathbf{x}=(x_1,x_2,x_3)$), q_j are components of the heat flux vector, $\Theta=(T-T_0)$ is the relative body temperature, ε_{ij}, σ_{ij} are the strain and stress tensors, u_i are the displacement components (these quantities are functions of $\mathbf{x}$ and the time t), and $T_0=\mathrm{const}$ is the body temperature in the natural state. The dots denote partial derivatives with respect to time, the subscript after the comma is the derivative with respect to the corresponding coordinate. Summation is over repeated subscripts.

Unlike the dynamic equations of the theory of elasticity and the non-stationary heat conduction equations, the generalized thermomechanics equations describe the mutual influence of the deformation and temperature fields and also take account of the finiteness of the heat propagation velocity. It is important to take these effects into account in any study of the qualitative behaviour of the solution. At the same time, in quantitative respects taking them into account does not result in any appreciable difference between the solutions and the solutions of the elasticity and heat conduction theories in many cases /2, 3/.

In view of this, we will assume that the terms $T_0\beta_{ij}\varepsilon_{ij}^{\cdot}$, $\tau q_j^{\cdot}$, $\beta_{ij}\Theta$ are small quantities of the order of ε $(0<\varepsilon\ll 1)$ and the solution of system (1.1) $\{q_j,\Theta,\sigma_{ij},\varepsilon_{ij},u_i\}$ differs from the function $\{q_j^\circ,\Theta^\circ,\sigma_{ij}^\circ,u_i^\circ\}$, which is a solution of the mutually uncoupled non-stationary equations of the theory of heat conduction and the dynamic equations of elasticity theory, by a quantity $O(\varepsilon)$

$$\overset{\circ}{q}_{j,j}+C_\varepsilon^\circ\Theta^{\circ\cdot}=0,\quad q_j^\circ+K_{ij}^\circ\Theta_{,i}^\circ=0,\quad \overset{\circ}{\sigma}_{ij,j}=\rho^\circ\overset{\circ}{u}_i^{\cdot\cdot} \tag{1.2}$$
$$\varepsilon_{ij}^\circ=\frac{1}{2}(\overset{\circ}{u}_{i,j}+\overset{\circ}{u}_{j,i}),\quad \sigma_{ij}^\circ=\overset{\circ}{C}_{ijkl}\overset{\circ}{e}_{kl},\quad i,j,k,l=1,2,3$$

Here we assume that $|C_\varepsilon-C_\varepsilon^\circ|$, $|K_{ij}-K_{ij}^\circ|$, $|\rho-\rho^\circ|$, $|C_{ijkl}-C_{ijkl}^\circ|$ are also of the order

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